Investigation of Lift-offset on Flight Dynamics Characteristics for Coaxial Compound Helicopters

Ye Yuan*
Nanjing University of Aeronautics and Astronautics, 210016 Nanjing, People's Republic of China

Douglas Thomson.†
University of Glasgow, Glasgow, Scotland, G12 8QQ, United Kingdom

and
Renliang Chen‡
Nanjing University of Aeronautics and Astronautics, 210016 Nanjing, People's Republic of China

The coaxial compound helicopters adopt the Lift-Offset (LOS) strategy to improve the rotor performance in high speed flight, which influences the flight dynamics characteristics of the vehicle. Thus, a flight dynamics model and an inverse simulation method are developed to assess the effect of LOS on this helicopter. The trim results demonstrate that LOS reduces the collective and the longitudinal cyclic pitch across the flight range, and it also adds the control input of the propeller collective in hover and lower speed forward flight. LOS control strategy reduces the power consumption and increases the maximum flight speed. Also, LOS control strategy is affected by the gross weight. Furthermore, the stability is dependent on LOS due to its effect on the rotor efficiency and flapping motion. From the controllability results, LOS brings about severe coupling between the rolling moment and the collective differential input. Lastly, the Pull-up & Push-over and the Transient Turn Mission-Task-Elements (MTEs) with different LOS control strategies are assessed with inverse simulation. The results show that a reasonable LOS control strategy could not only reduce the power consumption but also have a positive influence on the oscillation in control inputs during the manoeuvre.

* Ph.D. Candidate, National Key Laboratory of Science and Technology on Rotorcraft Aeromechanics, College of Aerospace Engineering, No. 29 Yudao Street. 210016. Jiangsu; sdyuanye@nuaa.edu.cn.
† Senior Lecturer, Aerospace Sciences; douglas.thomson@glasgow.ac.uk
‡ Professor, National Key Laboratory of Science and Technology on Rotorcraft Aeromechanics, College of Aerospace Engineering, No. 29 Yudao Street. 210016 Jiangsu; crlae@nuaa.edu.cn (Corresponding Author).

Nomenclature

A	=	system matrix
B	=	control matrix
K_β	=	equivalent flapping rigidity (N.m/rad)
$L_{\text{hub}}, M_{\text{hub}}$	=	rolling and pitching hub moment (N.m)
LOS	=	lift offset value
N_b	=	number of blade
R	=	rotor radius (m)
V	=	forward speed (m/s)
t	=	time (s)
Ω	=	rotor speed (rad/s)
γ	=	glideslope angle (°)
γ_{lc}	=	lock number
μ	=	advancing ratio
$\bar{\omega}_n$	=	non-dimensional first order flapping frequency
β	=	sideslip angle (°)
β_{1c}, β_{1s}	=	first order harmonic component of flapping angle (rad)
χ	=	track angle (°)

I.Introduction

THE coaxial compound helicopter has gained a lot of research interest in recent years due to its high-speed performance [1~2] and outstanding cruise-efficiency [3]. The efficiency of the coaxial rigid rotor (Advancing Blade Concept (ABC) rotor) can be significantly improved by applying what is termed Lift-Offset (LOS) strategy [4]. The conventional helicopter cannot have effective high-speed performance due to the dynamic stall problems on the retreating side of the rotor disc. When LOS strategy is adopted in coaxial compound helicopters, the lift-centre moves towards the advancing side and the retreating side is offloaded. Therefore, the stall problems can be prevented and the advancing side can reach its maximum lift-drag ratio, and consequently improves the performance of the coaxial compound helicopter in high speed flight [5].

LOS usually changes with the forward speed to optimize the rotor efficiency. In hover and low speed forward flight, LOS ought to be lower since the aerodynamic characteristics of the advancing and retreating sides are more alike. Therefore, offloading the retreating side results in extra profile drag [6]. Thus, LOS should increase with forward speed to avoid the dynamic stall effect and obtain a better rotor performance. However, LOS strategy is not only related to the rotor aerodynamic efficiency but also connected with the flight dynamics characteristics [7]. Firstly, the differential lateral cyclic, used to control the LOS of the rotor, is a unique redundant control for this type of helicopter. The input of the differential lateral cyclic alters the flapping characteristics of the coaxial rigid rotor and consequently influences the trim features, controllability, and stability [8]. In addition, LOS changes the rotor aerodynamic characteristics, which further complexes the handling qualities. Furthermore, using LOS strategy can also impact the manoeuvre performance, which may lead to additional pilot workload during the manoeuvre. Therefore, the LOS control strategy has a significant influence on the flight dynamics characteristics of the coaxial compound helicopter and there is a need for investigation this effect to a wider extent.

There has already been some research to date on the LOS characteristics for the coaxial compound helicopter, however most of them focus on aerodynamics and aeroacoustics features [4, 6-7, 9-17]. In the wind tunnel and flight test of XH-59A coaxial compound helicopter, the research was related to the flight dynamics characteristics [18-20]. During this flight test, the authors utilized the control phase angle to adjust LOS for a higher rotor loading in high speed flight. The flight test results showed that there is a significant improvement in rotor aerodynamics loading with re-scheduled LOS, however, the control phase angle reduced the rotor cyclic control power and consequently led to additional flight dynamics problem during the tests. During the design process of X2TD coaxial compound helicopter, the lateral cyclic pitch differential was introduced to control LOS as it do not affect the control power of the cyclic pitch[21-23]. In addition, a forward speed related LOS control strategy was proposed to optimize the rotor performance across the speed range. This strategy obtained from the wind tunnel experiment only related to the rotor aerodynamics and needs further investigation in terms of the flight dynamics characteristics. Ferguson has investigated the flight dynamics characteristics and performance of the coaxial compound helicopters [24-26], in which the LOS control strategy is developed based on the experiments [21-23]. Due to the lack of comparison between different LOS control strategies, the influence of LOS on the flight dynamics characteristics is still unclear. In conclusion, there is plenty of research focus on LOS of the coaxial compound helicopter though, but its effect on the flight dynamics characteristics still requires a further development. In addition, there is little research into the LOS control strategy in manoeuvring flight.

Considering the preceding discussion, this article first introduces the existing flight dynamics model and the inverse simulation method for the manoeuvre investigation. Then, the trim characteristics with respect to the different LOS strategies in various forward speed are analyzed. The power consumption of the coaxial compound helicopter is also obtained with various LOS values, and the power required results with different gross weight are set as comparisons to understand its influence on the LOS control strategy. Then, the typical stability derivatives including velocity stability, incidence stability, dihedral effect, and heading stability derivatives are assessed with different LOS strategy. In the controllability investigation, the on-axis and off-axis coupling control derivatives are obtained and the control derivatives produced by changing LOS are also calculated to further investigate its influence on the flight dynamics characteristics of the coaxial compound helicopter. Lastly, this article chooses the Pull-up & Push-over and Transient Turn MTE from the ADS-33E handling qualities specification to investigate the LOS effect on the manoeuvre characteristics.

II.Methodology

A. Flight Dynamics Model

The coaxial compound helicopter is a novel rotorcraft configuration. It utilizes the coaxial rigid rotor (known as advancing blade concept rotor) to guarantee the rotor performance in high speed flight. This helicopter configuration is usually equipped with an auxiliary tail-mounted propeller to provide the thrust at the high speed flight range. The rigidity of the coaxial rotor used in this helicopter configuration is much higher than others, which indicates that using the coaxial rotor to provide the thrust would drive the fuselage to tilt forward, and consequently increase the parasite drag of the fuselage and reduce the helicopter performance. Therefore, a propeller can be used to provide the thrust that the helicopter needs in trimmed and manoeuvring flight and guarantee the helicopter performance in high speed. The coaxial compound helicopter model is described by references [27, 28], and it is based on the XH-59A helicopter. The primary parameters of this helicopter are shown in Table. 1 [18-20, 27, 28].

Table. 1 XH-59A Helicopter Parameters

Parameter	Value
Rotor radius/m	5.49
Number of blades	3×2
Pretwist/(°)	-10
Rotor speed/(rad/s)	29.4-35.9
Taper ratio	2
Flapping frequency/Ω	1.4
Shaft spacing/m	0.77
Horizontal tail area/m^2	5.57
Vertical tail area/m^2	2.79
Takeoff mass/kg	5500
Lower Rotor position/m	(0.00,0.00,-0.89)
Centre of gravity/m	(0.00,0.00,0.00)
Horizontal tail position/m	(-6.80,0.00,0.20)
Vertical tail position/m	(-6.80,0.00,-0.50)

As the XH-59A helicopter utilized an auxiliary propulsion unit rather than a propeller to provide the thrust in high speed, this article uses a propeller instead, which is more in line with the development of coaxial compound helicopters in recent years. The parameters of the propeller are shown in Table.2 [25].

Table. 2 Parameters of Compound Propeller

Parameter	Value
Propeller radius/m	1.3
Propeller Rotor speed/(rad/s)	162
Pre-twist/°	-30
Solidity	0.2
Position/m	(-7.66,0.00,0.00)

The flight dynamics model of this coaxial compound helicopter has been verified with flight test data and simulation results [18, 25]. The model is composed of five parts: rotor, propeller, horizontal tail, vertical tail, and fuselage.

In the rotor part, a conventional disc-type model is used to calculate the forces and moments. The induced velocity model is based on the Pitt-Peters dynamic inflow model [29] and assumes the induced inflow of the lower rotor does not affect the upper rotor's ability to generate thrust, and the rotors are sufficiently close together that the wake from the upper rotor does not fully develop [25]. This assumption is based on the reference [25], and it has been proved that the assumption has acceptable precision because the coaxial rigid rotor usually features very stiff blades with a small separation distance between rotors. In addition, the rotor model ignores the pitching and lagging DOFs, assuming the flap motion has the most influence on the flight dynamics characteristics. To simulate the flapping motion more precisely, the model utilizes the equivalence method of the combination of equivalent flapping offset and flapping spring [30, 31]. An airfoil aerodynamic look-up table is utilized in load calculation of the rotors. The control strategy of this rotor is different from that of the conventional helicopter. The control equations for the coaxial rotor system are shown as follows:

$$\theta_U = (\theta_0 + \theta_{01}) - B_1 \cos(\psi_U + \Gamma) - (A_1 + \theta_{dc}) \sin(\psi_U + \Gamma) \tag{1}$$

$$\theta_L = (\theta_0 - \theta_{01}) - B_1 \cos(\psi_L + \Gamma) + (A_1 - \theta_{dc}) \sin(\psi_L + \Gamma) \tag{2}$$

where the subscripts of *U, L* are represented for the upper and lower rotor respectively; θ is the blade pitch; θ_0 is the collective pitch; θ_{01} is the differential collective pitch; ψ is the rotor plane relative azimuth angle; A_1 , B_1 are the lateral and longitudinal cyclic pitch; Γ is the control phase angle; θ_{dc} is the differential lateral cyclic pitch, which is used to control LOS.

The other parts of the coaxial compound helicopter flight dynamics model are captured in a similar way to other conventional helicopter models. The propeller part is similar to the rotor model except that there is no flapping motion DOF in the propeller. The fuselage model uses aerodynamics data from wind tunnel tests [32], in which force and moment coefficients of the wind tunnel test are dependent on the fuselage angle of attack and angle of sideslip. A 2-D representation of the horizontal and vertical tail using strip theory is incorporated into the model. The lift and drag coefficients can be obtained from a 2-D airfoil aerodynamics look-up table with given angle of attack and angle of sideslip. Also, a rudder deflection correction is added in the vertical tail aerodynamic model [33].

The flight dynamics model of the coaxial compound helicopter contains 21 DOFs, including 6 DOFs of the fuselage rigid motions, 6 DOFs of the induced velocities of the coaxial rotor, 6 DOFs of the upper and lower rotor flapping motions, and 3 DOFs of the propeller induced velocities. The state-space equations of the model can be expressed as:

$$\dot{\boldsymbol{x}} = f(\boldsymbol{x}, \boldsymbol{u}, t) \tag{3}$$

where $\boldsymbol{x} = [\boldsymbol{E}, \boldsymbol{F}, \boldsymbol{G}, \boldsymbol{H}]^T$, $\boldsymbol{E} = [u, v, w, p, q, r, \phi, \theta, \psi]^T$ represents the velocity, the angular velocity, and the Euler angle of the fuselage; $\boldsymbol{F} = [\beta_{L0}, \beta_{Lc}, \beta_{Ls}, \beta_{U0}, \beta_{Uc}, \beta_{Us}]^T$ is the flapping motion of the lower and upper rotors; $\boldsymbol{G} = [v_{L0}, v_{Lc}, v_{Ls}, v_{U0}, v_{Uc}, v_{Us}]^T$ is the induced velocity of the lower and upper rotors; $\boldsymbol{H} = [v_{p0}, v_{pc}, v_{ps}]^T$ is the induced inflow of the propeller. $\boldsymbol{u}=[\theta_0, A_1, B_1, \theta_{01}, \theta_{dc}, \theta_p, \beta_{ru}]^T$ is the control input of the coaxial compound helicopter. In order to calculate the stability and control derivatives, Eq. (3) is linearized [25] and written in state-space form as:

$$\dot{\boldsymbol{x}}_{linear} = A\boldsymbol{x}_{linear} + B\boldsymbol{u}_{linear} \tag{4}$$

where $\boldsymbol{x}_{linear}=[u, v, w, p, q, r, \phi, \theta]^T$ is the state vector in linearization; $\boldsymbol{u}_{linear}=[\theta_0, A_1, B_1, \theta_{01}, \theta_{\mathrm{dc}}, \beta_{ru}]^T$ is the control vector in linearization in this article. The state and control vectors are perturbations from the trimmed state.

B. Trim Strategies

In this article, the influence of LOS on the trim characteristics, performance, stability, controllability, and the handling qualities will be analyzed. The starting point of the analysis is the trim results. For the coaxial compound helicopter, except for the LOS control strategy which would be discussed individually below, there are three redundant control inputs that should be mentioned: the propeller collective, rotor speed, and the rudder deflection.

The auxiliary propeller can be used to provide thrust in high speed flight to offload the rotor and improve the helicopter performance. The propeller thrust is, therefore, an additional unknown trim variable in the trim process. Thus, a fuselage pitch attitude schedule [25] is used to trim the propeller collective, which is shown in Fig.1.

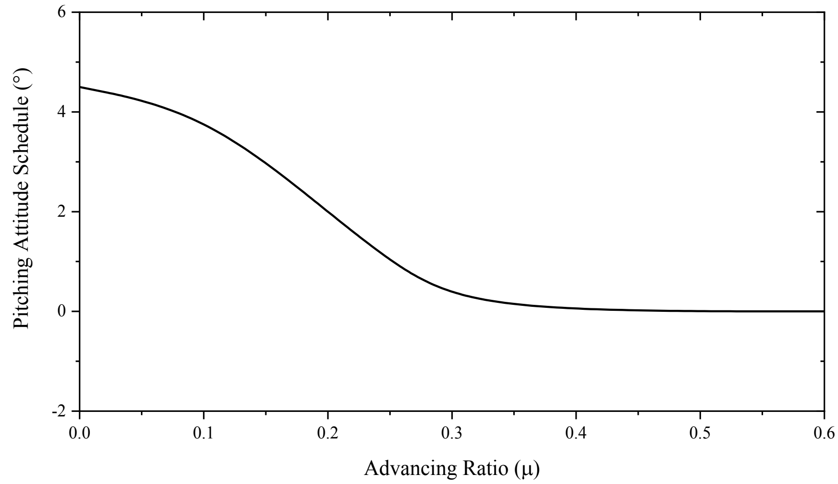


Fig. 1 Pitching Attitude Pre-determined Schedule

The coaxial rotor rotational speed should be slowed down to avoid the compressibility effect at the advancing blade tip in high speed flight. Therefore, a pre-scheduled rotor speed is set, which is [17]:

$$\Omega=\begin{cases} 35.9, \quad V<70m/s \\ 35.9-\dfrac{6.5(V-70)}{30}, V \geq 70m/s \end{cases} \tag{5}$$

Both the rudder deflection and the differential collective pitch could be used for heading control. However, the control power of the differential collective decreases dramatically with forward speed increases [34]. Thus, the differential collective is washed out between 20m/s and 40m/s and reintroduced when decelerating from 40m/s to 20m/s [18], which can be concluded as follow:

$$\begin{cases} \theta_{01}=\theta_{\text{pedal}}, V<20 \\ \theta_{01}=(2-0.05V)\theta_{\text{pedal}}, 20<V\leq 40 \\ \theta_{01}=0, V\geq 40 \end{cases} \tag{6}$$

$$\begin{cases} \beta_{ru}=0, V<20 \\ \beta_{ru}=0.05(V-20)\theta_{\text{pedal}}, 20<V\leq 40 \\ \beta_{ru}=\theta_{\text{pedal}}, V\geq 40 \end{cases} \tag{7}$$

Equations 6 and 7 are used to define the heading control strategy for this helicopter.

C. LOS Strategy

A coaxial rotor with LOS can attain good efficiency by operating with more lift on the advancing side than the retreating side. The extent of the lift offset can be evaluated by the LOS parameter, which is defined as:

$$LOS = \frac{\Delta M_x}{TR} \tag{8}$$

where ΔM_x is the upper rotor rolling moment; T is the total rotor thrust. Previous studies have indicated that the LOS value should vary with the forward speed to maintain the rotor efficiency. Too low LOS forces the retreating area to provide more lift than its capability and therefore increases the profile drag, and too high LOS also results in the degradation of the rotor performance.

According to reference [21], a velocity related equation can be set to determine the LOS value at various speeds, which is:

$$LOS = 0.00002V^2 \tag{9}$$

The LOS strategy of Eq. (9) is derived from the experiment from reference 21, and Ferguson [24] concludes this strategy based on these results. The LOS strategy is designed to improve the flight dynamics characteristics and performance across the flight range. This strategy does not consider the effect of the rotor speed change. However, the change of the rotor rotational speed should be considered during the LOS control strategy design as it alters the dynamic pressure difference between the advancing and retreating sides of the rotor, which would cause more serious dynamic stall. Therefore, based on the rotor tip speed in the original setting (around 200 m/s), the modified LOS strategy is obtained:

$$LOS = 0.00002v_f^2 = 0.8(\frac{v_f}{\Omega R})^2 = 0.8\mu^2 \tag{10}$$

Based on Eq. (10), the rotor rotational speed can be taken into account during the LOS control strategy. Thus, Eq. (10) is used as the comparison in flight dynamics investigation.

D. Inverse Simulation

Combined with the MTE defined in ADS-33E [35], the inverse simulation method is a convenient way to assess the manoeuvrability of the helicopter. This method is explained widely in the literature by various authors [36-40]. Therefore, only a brief overview of this method is shown in this article.

The forward time response solution of the rotorcraft is readily available when the flight dynamics model is constructed. The inverse simulation can be represented as a "trim process" with respect to each time steps through a predefined trajectory or manoeuvre. At the new time increment, the control input must be varied to ensure the correct flight path, which is given by the requirement of the MTE or other manoeuvres.

To process the inverse simulation algorithm, this article executes the following steps with respect to the characteristics of the coaxial compound helicopter:

1). Calculate the trim control input

The trim states correspond to steady level flight with the body accelerations and the attitude rates equal to zero, which is the initial point of the MTE manoeuvres. The trim variables of the conventional helicopters are equal to the trim target equations. However, the coaxial compound helicopter has redundant control inputs. Thus, the trim and LOS strategies mentioned above are utilized here to determine the initial point of MTE.

2). Define the manoeuvre

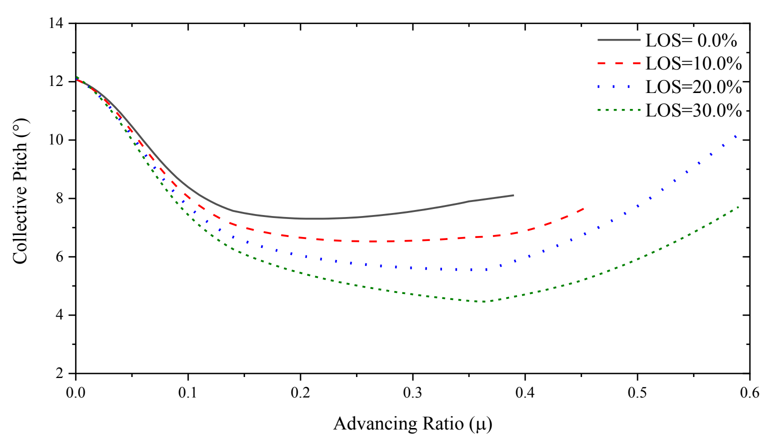
The manoeuvre can be defined simply by polynomial representations of position or other flight path variables and is then discretized into a series of discrete time points. The redundant control of the coaxial compound helicopter may affect the definition of the manoeuvre as they may need additional polynomials to obtain their control inputs with respect to the time step. In this article, the strategies of redundant control inputs are determined based on the MTE features and the LOS effect analysis.

3). Calculate the control vector

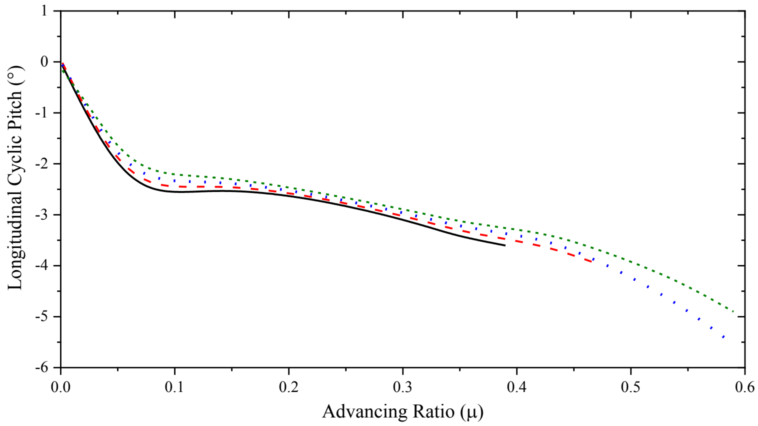
This inverse simulation model uses a Newton-Raphson method to calculate the control inputs to maintain the helicopter's states in accordance with the manoeuvre mathematical description. This process is repeated throughout each time-step until the manoeuvre has been completed.

III.Trim Analysis

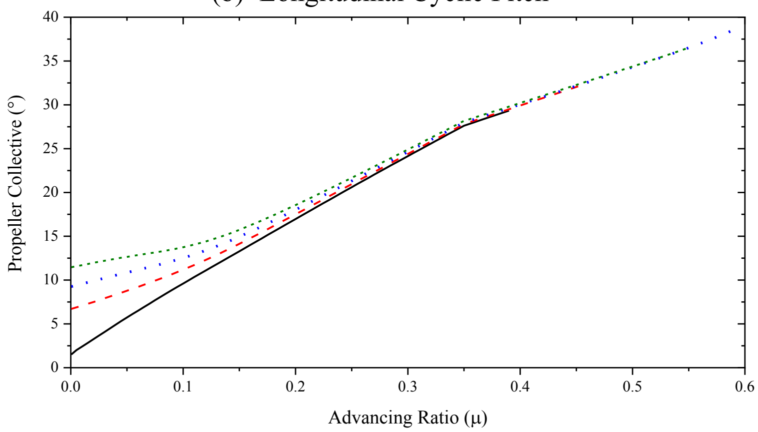
The trim results at various advancing ratios with LOS of 0%, 10%, 20%, and 30% are shown in Fig. 2.



(a) Collective Pitch



(b) Longitudinal Cyclic Pitch



(c) Propeller Collective

Fig. 2 Trim Results with Various LOS

Fig. 2 demonstrates that LOS affects the trim characteristics of the coaxial compound helicopter. As indicated in Fig. 2(a), the collective pitch decreases with LOS increases, especially in high speed flight. At this flight range, most of the area of the retreating side is in reverse flow and its dynamic pressure is relatively low. Increasing LOS value means the incidence of advancing side increases, and consequently the rotor would provide extra rotor lift to the helicopter and a further drop occurs in the trim collective pitch. Also, it should be mentioned that the trend of the collective pitch and other results are changed when the advancing ratio is around 0.35 due to the change of the rotor rotational speed.

According to Fig. 2(b), when LOS increases, the pilot needs to slightly pull backward the cyclic stick to balance the rotorcraft. LOS reduced the collective pitch results, and consequently the coning angle of the rotor drops. Therefore, the upward pitching moment decreases, which needs extra longitudinal cyclic pitch to balance the pitching moment. In addition, the difference in longitudinal cyclic pitch is more significant when the advancing ratio is beyond

0.35 due to the change in rotor rotational speed. The change of rotor rotational speed results in the alteration of the flapping frequency, which reduces the flapping response lag angle. On the other hand, the control phase angle is constant. Therefore, it causes the differential lateral cyclic pitch to provide extra pitching moment to the helicopter, which brings about the change in the longitudinal cyclic pitch results.

As indicated in Fig. 2(c), LOS would mainly increase the trim propeller collective results in hover and low speed forward flight range. In other words, LOS alters the drag produced by the coaxial rotor at this flight range. In fact, the differential lateral cyclic pitch makes the airfoil drag of the advancing and retreating areas different. This phenomenon is demonstrated in Fig. 3.

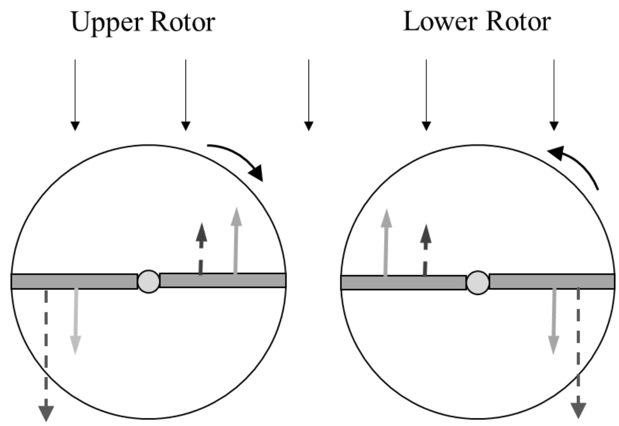


Fig. 3 The Influence of LOS on the Rotor Drag

When LOS is around 0, the incidences of the advancing and retreating sides are similar. Thus, the drag (grey solid line in Fig. 3) on rotor disc are counteracting. However, when LOS is more than 0, the incidence of the advancing side is more than that of the retreating side. This increases the drag of the advancing side (black dash line in Fig. 3), which needs the extra propeller collective to balance it. When the forward speed increases, this effect tends to be diminished because the dynamic pressure of the advancing and retreating sides are different and the difference in profile drag is no longer significant. Also, the parasite drag from the fuselage and other parts of the helicopter becomes the major part for the propeller to overcome. This feature accounts for the propeller collective trim input with different LOS values is almost invariable in high speed flight. In addition, as the distance between the propeller and the centre of gravity is close and the cyclic pitch control derivatives of the rigid rotor are relatively high, the longitudinal moment produced by this extra drag does not significantly influence the longitudinal cyclic pitch trim results.

IV.Performance Analysis

The power consumption results with LOS of 0%, 10%, 20%, and 30% at various advancing ratios are shown in Fig.4.

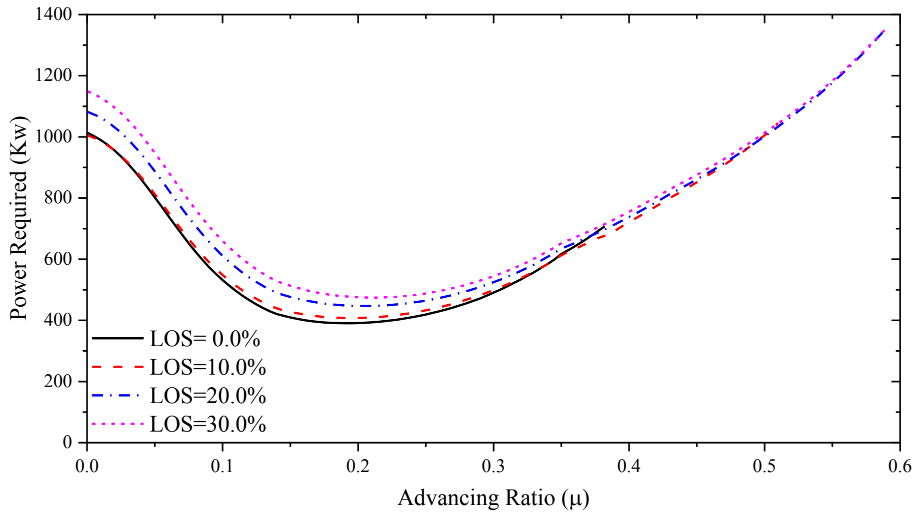
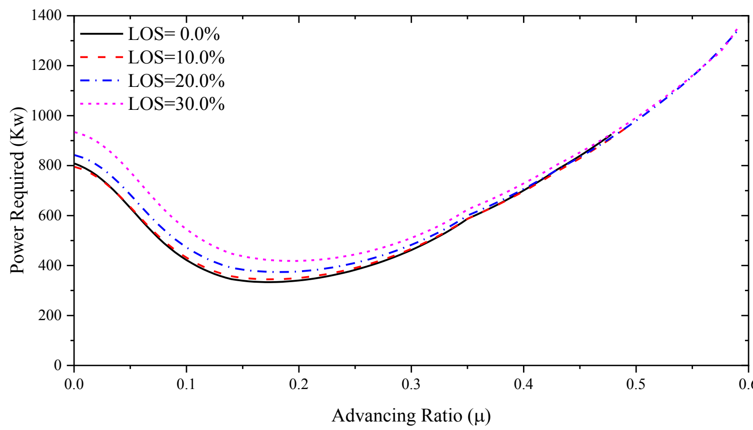


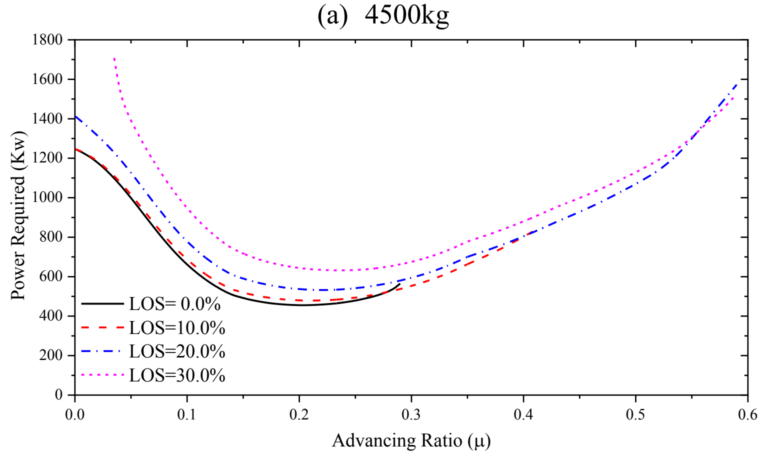
Fig.4 Power Required Comparison

According to the power required comparison with various LOS values, LOS should increase with forward speed, which is in line with the original setting in Eq. (9) and Eq. (10). In hover and low speed forward flight, increasing LOS reduces the rotor efficiency as the dynamic pressure on the retreating side is still high and it also leads the angle of attack at the advancing side to exceed the optimal range of the airfoil. As the forward speed increases, the trim state cannot be obtained when LOS is relatively low. The dynamic pressure at the retreating side is low and most of its area is experiencing reverse flow, and the rotor system needs the advancing side to provide more lift, which forces the LOS to increase to maintain the trimmed state. Also, LOS mainly affects the power consumption at mid forward speed range. The propeller occupies the most of the power required in high speed flight. Thus, LOS does not influence the overall power consumption to a large extent as the rotor has been offloaded at this time. However, the LOS still has a significant influence in high speed flight. According to Fig. 4, if the LOS is relatively low, the coaxial compound helicopter cannot obtain the trimmed state in high speed as the rotor performance is too low to provide the lift that the helicopter needs. The related analysis and discussion would be shown later in this paper.

Based on the research from others [5, 7, 10], the relationship between LOS and helicopter performance could be affected by other factors, such as the gross weight of the helicopter. Thus, the power consumptions in 4500kg and 6500kg gross weights are calculated (The original gross weight is 5500 kg). These results are shown in Fig. 5.



(a) 4500kg



(b) 6500kg

Fig.5 Power Required Comparison in Different Gross Weight

As indicated in Fig.5, the gross weight would influence the LOS effect on the maximum flight speed and power consumption of the coaxial compound helicopter. Firstly, when the gross weight is lower, the trim state can be accomplished with less LOS value in higher flight speed as the lift that the coaxial rotor system needs to provide is dependent on the gross weight. In addition, the helicopter cannot trim with a relatively high LOS in hover and low speed forward flight with a higher gross weight, which is also due to the reduction of the rotor performance. When LOS is relatively high with lower forward speed, the efficiency of the rotor is reduced to a large extent. On the other hand, the increase of the gross weight adds on the rotor payload, which is harder for the rotor system to provide sufficient thrust.

Also, the influence of LOS on power consumption is more significant when the gross weight increases. Comparing with the results in Fig. 4, the optimal LOS setting for minimum power consumption changes with gross weight. In other words, it demonstrates that the original setting would be no longer optimized when the gross weight changed.

Meanwhile, it should be mentioned that although the LOS effect on the power required is relatively low in high speed flight, its value would decide whether the coaxial compound helicopter has sufficient rotor performance to obtain the trim status at this flight range. In order to fully investigate the effect of LOS on the helicopter, the LOS envelope is calculated, which indicates the relationship between the LOS value and the maximum forward speed in different gross weights. The result is shown in Fig. 6.

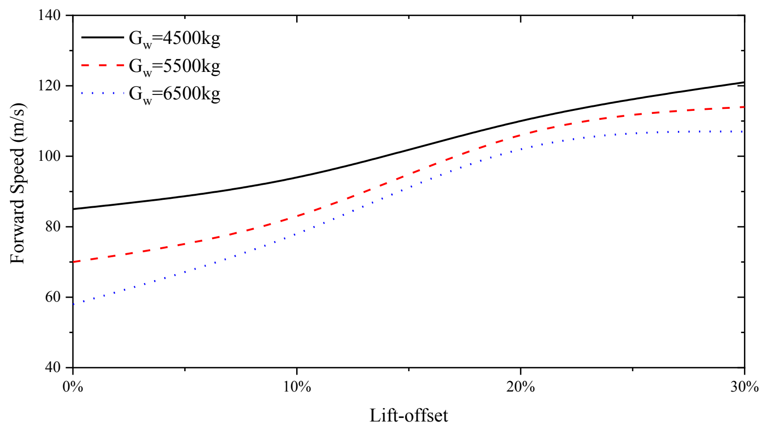
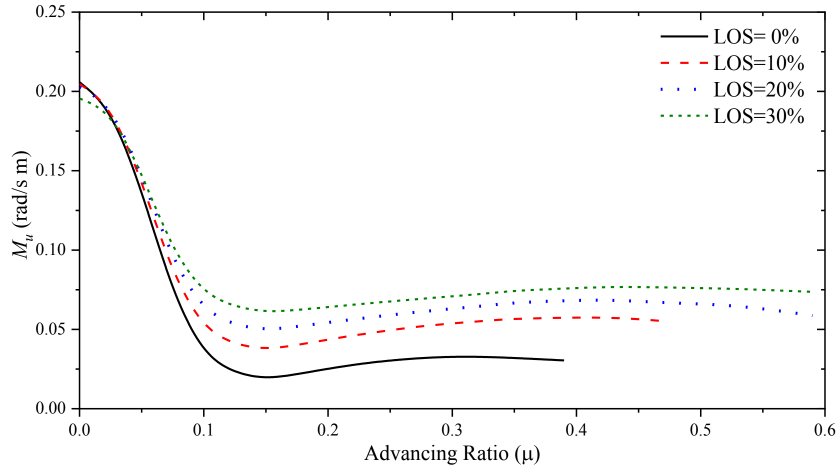


Fig.6 The Relationship between Lift-offset and Maximum Forward Speed with Different Gross Weights

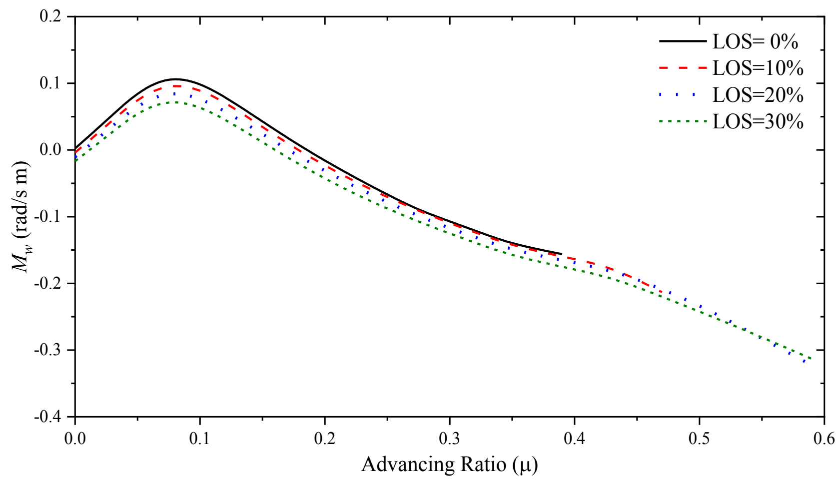
According to Fig. 6, when the LOS value is relatively low, the coaxial compound helicopter cannot maintain a trimmed state in high speed flight due to the effect of dynamic stall in the retreating side. With the increase of the LOS value, the maximum speed can be improved. Moreover, the gross weight also plays an important role in the LOS envelope, the increase in gross weight would reduce the LOS envelope. This is because more collective pitch is needed with a heavier gross weight, and makes the dynamic stall phenomenon more serious.

V.Stability Analysis

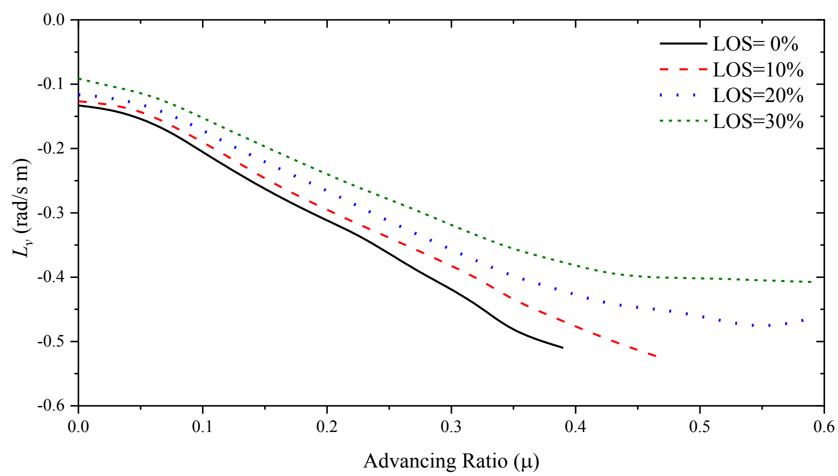
This article investigates the typical stability derivatives with various advancing ratios in different LOS values. The derivatives include the velocity stability derivative, incidence stability derivative, dihedral effect derivatives, and heading stability derivatives. The results are shown in Fig. 7.



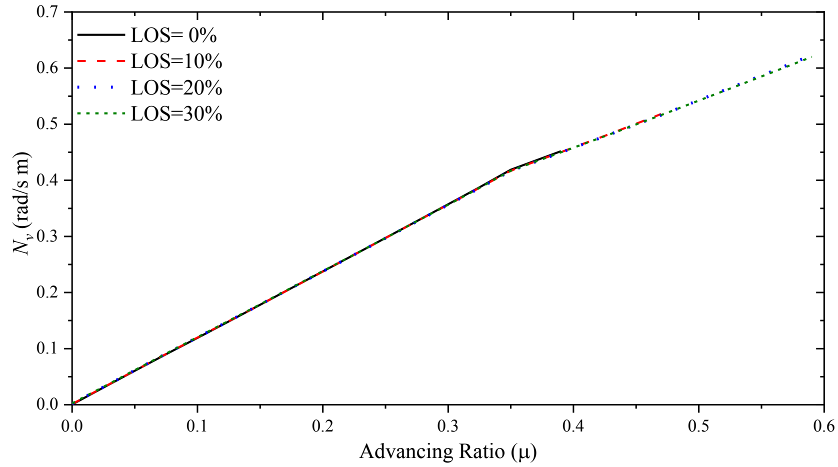
(a) Velocity Stability Derivatives



(b) Incidence Stability Derivative



(c) Dihedral Effect Derivatives



(d) Heading Stability Derivatives

Fig. 7 Typical Stability Derivatives with Different LOS

The influence of LOS on stability can be divided into two aspects. Firstly, LOS is controlled by differential lateral cyclic pitch, which affects the rotor stability by altering the flapping motion. Secondly, LOS influences the rotor performance, which means it changes the overall rotor efficiency. This parameter influences the values of the stability derivatives produced by the rotors.

One of the derivatives most likely to be influenced by lift offset is the velocity stability derivative, M_u. Due to the flapping motion characteristics, the extra input of lateral cyclic (due to LOS) on each rotor disc influences this stability derivative. Also, this effect can be understood from the view of LOS. The increase of LOS means that more lift is provided by the advancing side of the rotor disc. On the other hand, the reverse flow area cannot provide much velocity stability since the blade element in this area is mostly in the stall, which is only slightly influenced by the velocity perturbation. Thus, the LOS value means the helicopter tends to use the advancing side to provide the lift. This results in the rotor more sensitive to the forward perturbation velocity.

The incidence stability M_w is positive (unstable) in hover and low speed forward flight due to the rotor features, and it becomes stable in high speed because the incidence stability provided by the horizontal tails increases with forward speed. As for the coaxial compound helicopter, a majority of the rotor stability derivatives come from the hub moment as it has higher flapping rigidity. The longitudinal and lateral hub moments can be calculated in terms of the flapping angles as follow [30]:

$$L_{hub} = -N_b \frac{K_\beta}{2} \beta_{1c} \tag{11}$$

$$M_{hub} = -N_b \frac{K_\beta}{2} \beta_{1s} \tag{12}$$

where L_{hub} and M_{hub} is the hub moment of rolling and pitching axes. The incidence stability from the rotor is related to LOS due to the change of lock number. The derivatives of the flapping angle with respect the inflow ratio λ ($\lambda = w_h / \Omega R$) are shown as follow [41]:

$$\begin{cases} (\bar{\omega}_{\mathrm{n}}^{\ 2}\text{-}1)\dfrac{\partial \beta_{1c}}{\partial \lambda} + [\dfrac{\gamma_{lc}}{8}(1+\dfrac{1}{2}\mu^2)]\dfrac{\partial \beta_{1s}}{\partial \lambda} = 0 \\ (\bar{\omega}_{\mathrm{n}}^{\ 2}\text{-}1)\dfrac{\partial \beta_{1s}}{\partial \lambda} + [\dfrac{\gamma_{lc}}{8}(\dfrac{1}{2}\mu^2 - 1)]\dfrac{\partial \beta_{1c}}{\partial \lambda} = -\dfrac{\gamma_{lc}}{4}\mu \end{cases} \tag{13}$$

Comparing the incidence stability results with the power consumption results in Fig.4, the influence of LOS on the incidence stability is reverse to the rotor efficiency. When the rotor is relatively efficient in terms of the power consumption, the incidence stability tends to be less stable. This is because the lock number directly affects these stability characteristics.

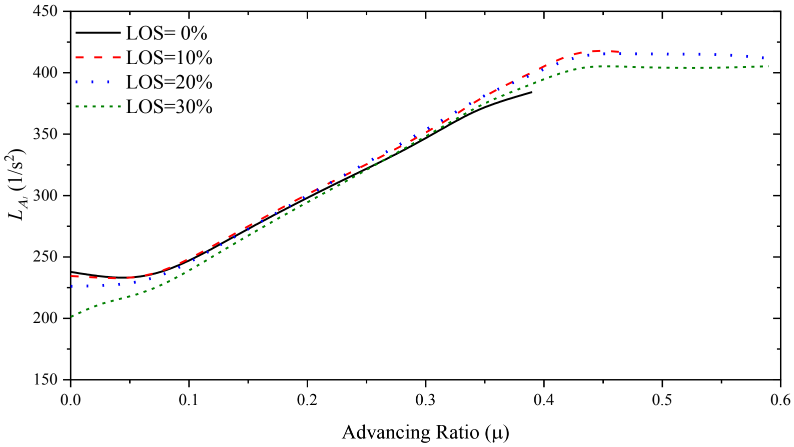
Also, LOS alters the lateral stability characteristics, especially the dihedral effect. The dihedral effect L_v is stable in most case with different LOS, but LOS influences this stability derivatives significantly. According to the trim analysis, the LOS effect is less influential on the longitudinal cyclic pitch, but it reduces the collective to a large extent. As a whole, this causes the incidence at 0 degrees and 180 degrees azimuth angle on the rotor disc decreases, which reduces the dihedral effect stability. On the other hand, LOS has little impact on the heading stability derivatives N_v because this stability derivative is mostly provided by the vertical tail.

VI.Controllability Analysis

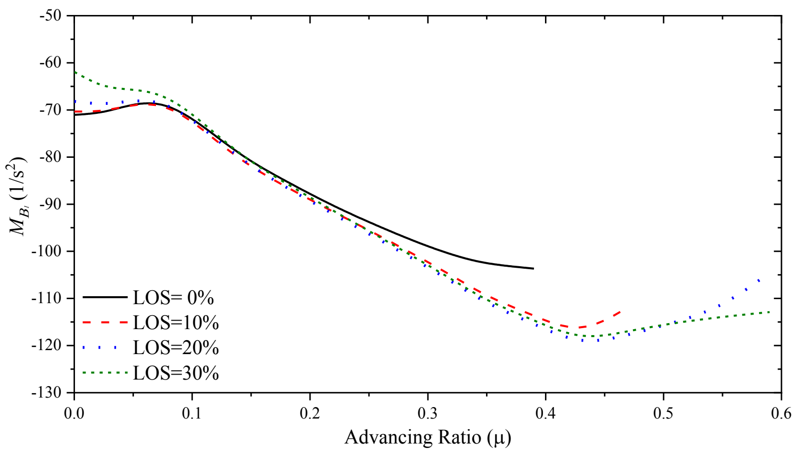
In this article, the controllability investigation can be divided into two aspects: first is to analyze the influence of LOS on the control derivatives, including the on-axis and off-axis coupling control derivatives. The second aspect is to investigate the control derivatives in terms of the differential lateral cyclic pitch. The aim of this part is to find whether the LOS change has any additional impact on the control characteristics of coaxial compound helicopters.

A. Control Derivatives

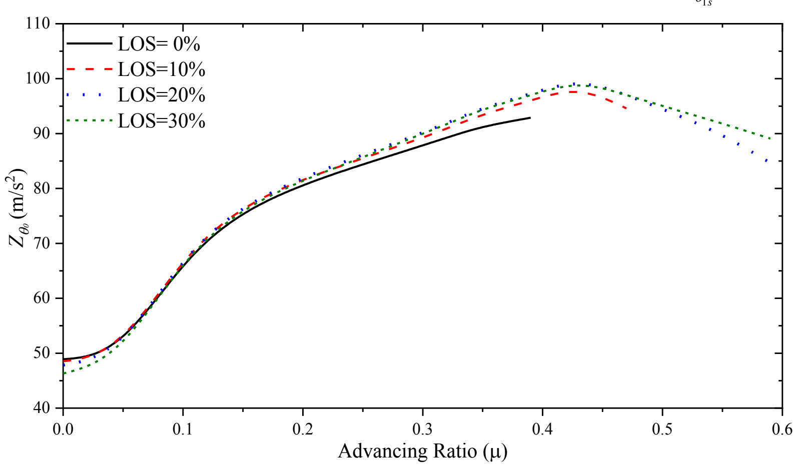
The on-axis control derivatives with different LOS with respect to the advancing ratio are shown in Fig. 8. It should be mentioned that the heading control derivatives are replaced with the collective differential control derivatives. This is because the rudder deflection derivatives are produced by the vertical tail, which has no significant connection with the LOS control strategy.



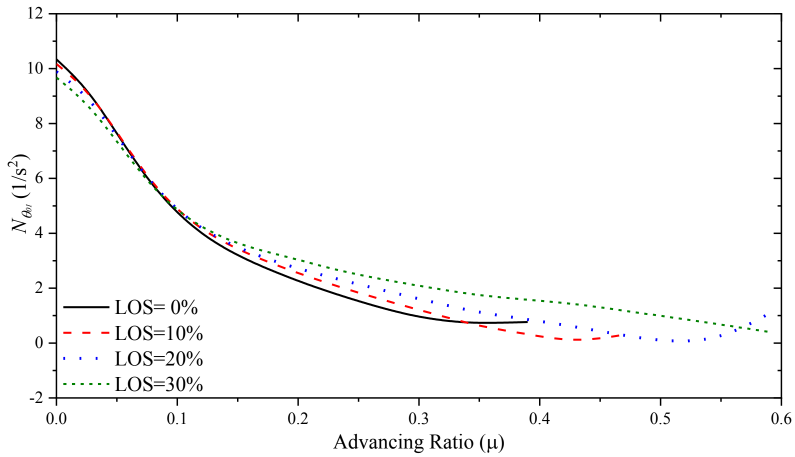
(a) Lateral Cyclic Pitch Control Derivatives (L_{A_1})



(b) Longitudinal Cyclic Pitch Control Derivatives($M_{\theta_{1s}}$)



(c) Collective Pitch Control Derivatives (Z_{θ_0})



(d) Collective Differential Control Derivatives ($N_{\theta_{01}}$)

Fig. 8 On-axis Control Derivatives

Fig.8 shows that the control characteristics of the coaxial compound helicopter are similar with the conventional helicopter except that longitudinal and lateral control derivatives increase with forward speed, the results are in accordance with the related results from other researchers [25]. The control derivatives (flapping angles) with respect to on-axis cyclic pitch can be approximately as follows, which is deduced based on the flapping equations [40]

$$\frac{\partial \beta_{1c}}{\partial A_1}=\frac{16\gamma(\bar{\omega}_{\rm n}{}^2\text{-}1)(\mu^2+2)}{256(\bar{\omega}_{\rm n}{}^2\text{-}1)^2+\gamma^2(4-\mu^4)}\cos(\Gamma)+\frac{\gamma^2(4-\mu^4)}{256(\bar{\omega}_{\rm n}{}^2\text{-}1)^2+\gamma^2(4-\mu^4)}\sin(\Gamma) \tag{14}$$

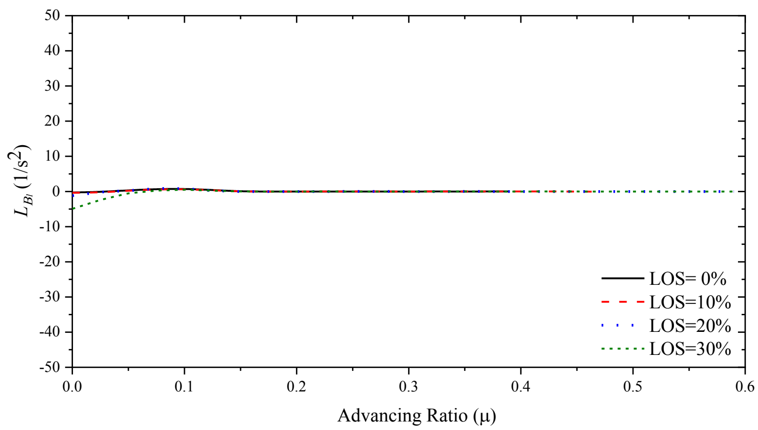
$$\frac{\partial \beta_{1s}}{\partial B_1}=\frac{16\gamma(\bar{\omega}_{\rm n}{}^2\text{-}1)(3\mu^2+2)}{256(\bar{\omega}_{\rm n}{}^2\text{-}1)^2+\gamma^2(4-\mu^4)}\cos(\Gamma)+\frac{\gamma^2(\mu^2+2)(3\mu^2+2)}{256(\bar{\omega}_{\rm n}{}^2\text{-}1)^2+\gamma^2(4-\mu^4)}\sin(\Gamma) \tag{15}$$

With the mathematical analysis on Eqns. (14, 15), when the flapping frequency is 1.4, these derivatives will increase with forward speed due to the higher flapping frequency. The difference from the conventional rotor in terms of these derivatives is due to the flapping frequency and the control phase angle. Meanwhile, the differential collective control derivatives are relatively low in high speed because of the rotor reverse flow area and the rotor offload at this flight range [33].

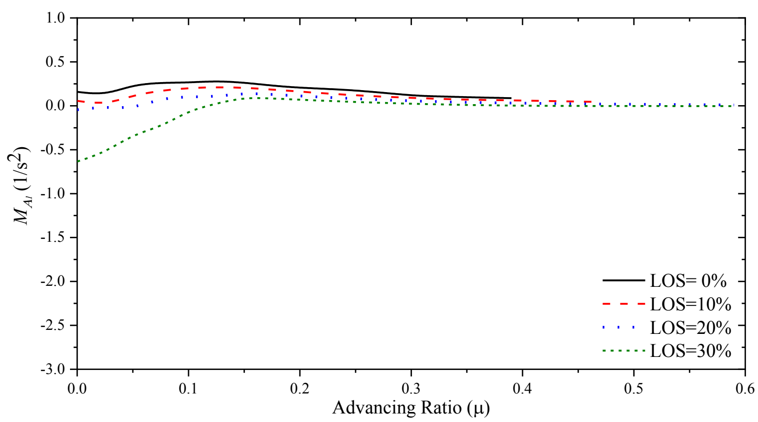
LOS could influence these control derivatives. Based on the results in Fig.8 and the power consumption in Fig.4, the on-axis control derivatives is dependent on the rotor efficiency (the lock number). In other words, at a given forward speed, LOS with a relatively high rotor performance would have larger on-axis control derivatives in different control inputs.

B. Coupling Derivatives

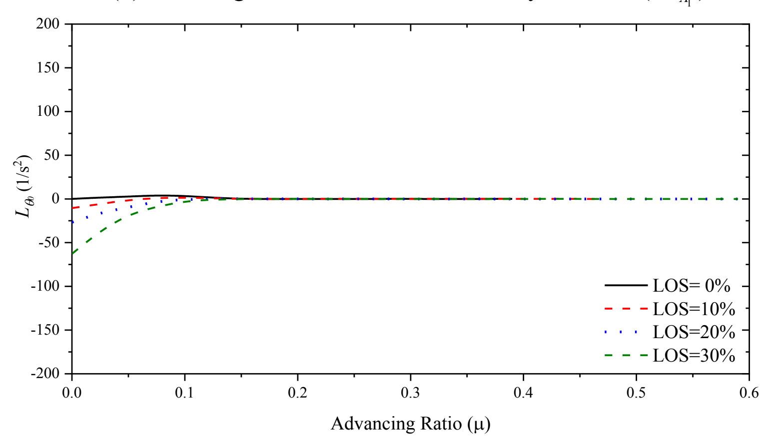
LOS influences the off-axis control coupling of the coaxial compound helicopter, and in order to investigate this effect, the typical coupling control derivatives $L_{B_1}, M_{A_1}, L_{\theta_0}, M_{\theta_0}, L_{\theta_{01}}$ are obtained in Fig.9.



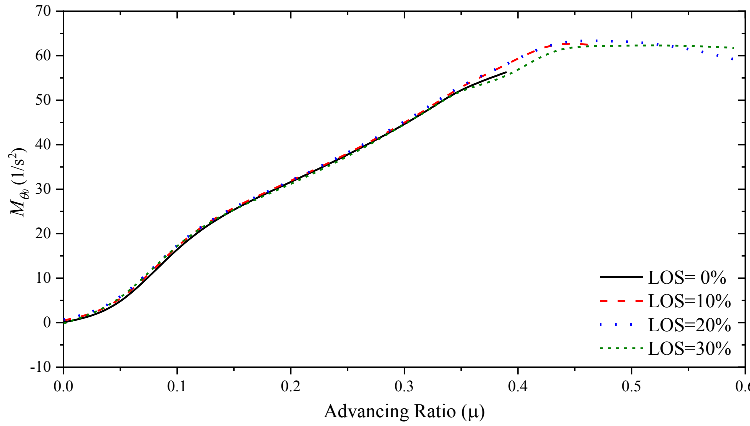
(a) Rolling Moment Due to Longitudinal Cyclic Pitch (L_{B_1})



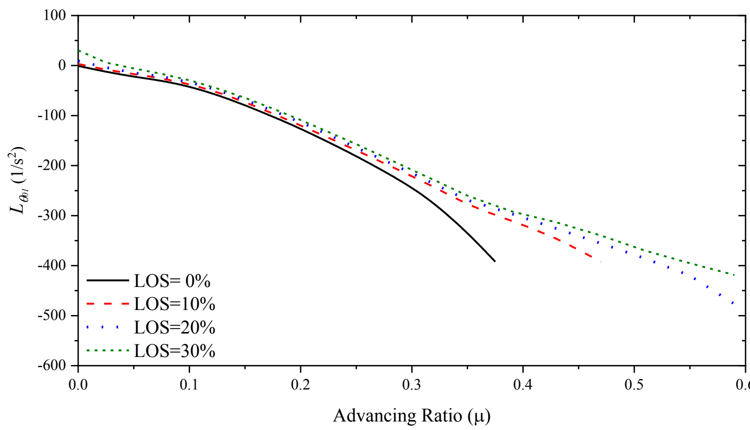
(b) Pitching Moment Due to Lateral Cyclic Pitch(M_{A_1})



(c) Rolling Moment Due to Collective Pitch (L_{θ_0})



(d) Pitching Moment Due to Collective Pitch (M_{θ_0})



(e) Rolling Moment Due to Collective Differential ($L_{\theta_{01}}$)

Fig. 9 Off-axis Coupling Control Derivatives

The off-axis coupling control derivatives for cyclic pitch are higher in hover and low speed forward flight, then reduce in high speed flight, as shown in Fig. 9(a) and Fig. 9(b). The aerodynamic interference of the coaxial rotor couples with the flight dynamics characteristics in hover and low speed forward flight to a large extent. Thus, it would lead to the imbalance in the coupling control moment in longitudinal and lateral axis and further causes the coupling phenomenon. The aerodynamic interference also causes the coupling between rolling moment and the collective pitch input. With forward speed increases, the aerodynamic interference is diminished, and consequently the coupling phenomenon becomes lower.

As demonstrated in Fig. 9(c) and Fig. 9(d), the coupling derivatives of the pitching moment to the collective pitch input increase with forward speed. According to the flapping motion characteristics, the approximately longitudinal flapping angle derivatives with respect to collective pitch are also deduced based on the flapping equations of the rotors [41], which is shown as follow:

$$\frac{\partial \beta_{1c}}{\partial \theta_0} = \frac{16\mu\gamma^2(\mu^2+2)}{3\times(256(\bar{\omega}_n{}^2\text{-}1)^2+\gamma^2(4-\mu^4))} \tag{16}$$

Based on Eq. (16), the derivatives increase with forward speed (μ), and it is close to zero when the helicopter is in hover. In addition, the lock number also adds the coupling derivatives, which means LOS increases the coupling between the pitching moment and collective pitch input.

On the other hand, the collective pitch is less influential on the rolling moment. This is because the rolling moment of the upper and lower rotors are nearly in balance, and this coupling only occurs in hover and low speed forward flight due to the aerodynamic interference.

Fig. 9(e) shows the significant coupling on rolling moment in terms of differential collective input. When the collective differential takes effect, the rolling moment of the upper and lower rotors is different because of LOS, which is illustrated in Fig.10.

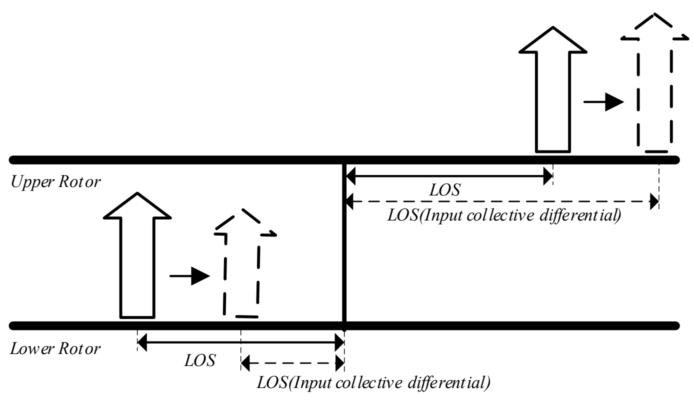
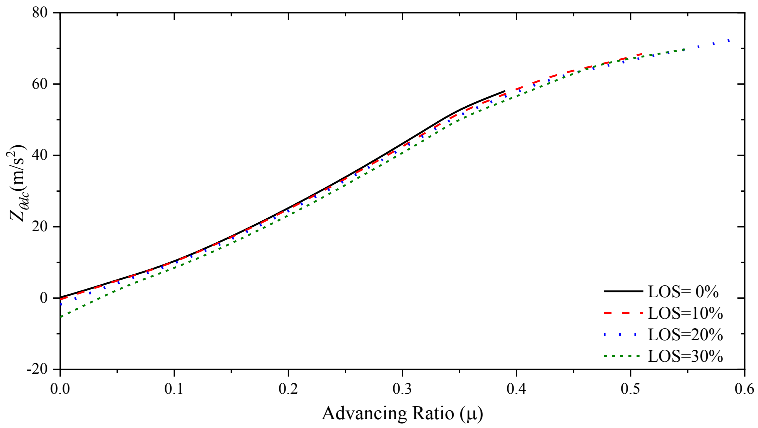


Fig.10 The Effect of LOS on Lateral-collective Differential Coupling

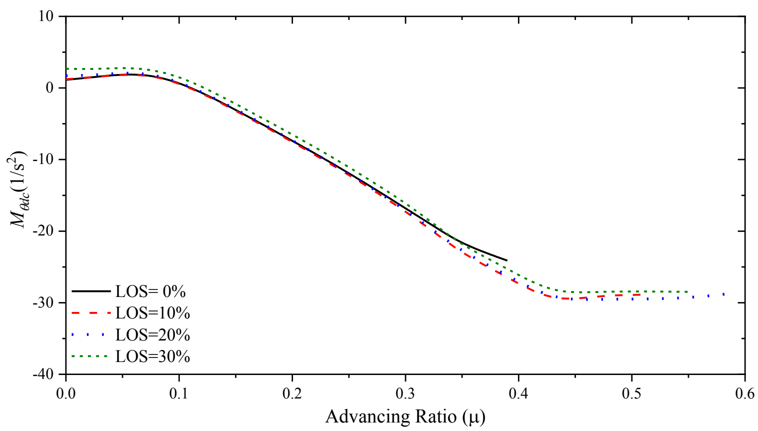
When the collective differential input is close to zero (the solid arrow in Fig. 10), LOS on the upper and lower rotors are similar to each other. The coaxial rotors are in balance in terms of the rolling moment. Once there is an input of the collective differential (the dash arrow in Fig. 10), the lift produced by the upper and lower rotor are different, and LOS of the upper and lower rotor also change separately. Both effects lead to a significant rolling moment to the helicopter and consequently cause the coupling phenomenon when the collective different is adopted.

C. LOS Derivatives (Control Derivative of Differential Lateral Cyclic)

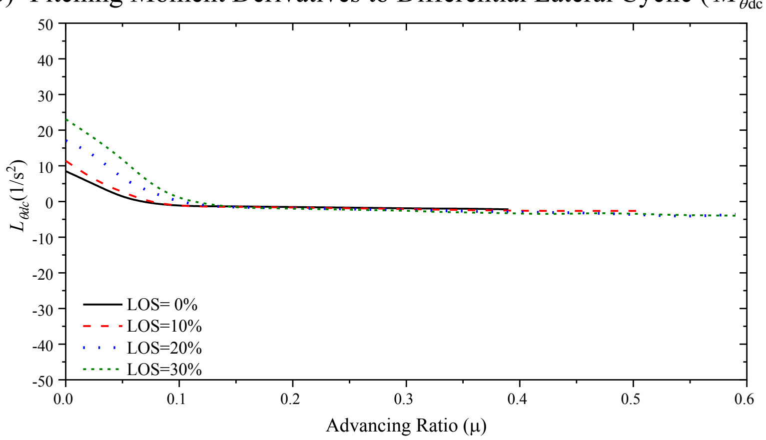
The derivatives of differential lateral cyclic (used to control LOS) on pitching moment, rolling moment, and the vertical force are investigated with different LOS in various advancing ratios. It aims to analyze the influence of LOS on the control characteristics of the coaxial compound helicopter. The results are shown in Fig. 11



(a) Vertical Force Derivatives to Differential Lateral Cyclic ($Z_{\theta\mathrm{dc}}$)



(b) Pitching Moment Derivatives to Differential Lateral Cyclic ($M_{\theta\mathrm{dc}}$)



(c) Rolling Moment Derivatives to Differential Lateral Cyclic ($L_{\theta\mathrm{dc}}$)

Fig.11 Typical Control Derivatives with Respect to Differential Lateral Cyclic

As illustrated in Fig.11, LOS influences the Z-direction force and pitching moment of the helicopter, and this influence becomes more obvious as forward speed increases. In addition, when the forward speed increases, the dynamic pressure of the advancing side is higher and the increase of lift on advancing side is larger than the decrease of the retreating side, which also affects the pitching moment due to the flapping feature. However, it hardly affects the rolling moment as the rolling coupling produced by the upper and lower rotor are equal.

VII. Manoeuvre Characteristics Investigation (MTE Assessment)

The preceding analysis demonstrates the influence of LOS on the flight dynamics characteristics of coaxial compound helicopters, however, it is also important to ensure that the coaxial compound helicopter can achieve its operational goals with the given LOS control strategy. Also, the rotor control inputs and flight environment change dramatically in aggressive manoeuvres, and therefore it is necessary to investigate the influence of LOS in manoeuvring flight. In addition, the power consumption during the manoeuvre also needs to be assessed in this article as LOS has the potential to reduce power required and therefore improves the ability of the vehicle in manoeuvring flight.

To fully investigate the influence of LOS on manoeuvring flight, this article uses the inverse simulation method to study Pull-up & Push-over and the Transient-Turn MTEs defined by ADS-33E-PRF [35]. These two MTEs allow consideration of the handling qualities of all axes. The velocities of these MTEs are around 60m/s, which is the mid to high flight speed range of the helicopter.

A. Pull-up & Push-over MTE

The objective of the Pull-up & Push-over MTE is to assess the handling qualities mainly in longitudinal and vertical channels. The mathematical description of this MTE has been explained widely in other literatures [42-43]. Therefore, only a brief overview of this method is shown in this article.

During the Pull-up & Push-over MTE, the helicopter starts in the trimmed condition at a flight speed equal to 120kt (approximately 60m/s). Then, it is required to achieve a positive normal load factor at given time (1s for level 1) and maintain this for a given period (2s for level 1). After that, the helicopter needs to transition to obtain a negative load factor (2s for level 1) and keep this load factor for a while (2s for level 1). Finally, the helicopter should recover to level flight as quickly as possible. The normal load factor n_z is a measure of vertical acceleration and is defined as:

$$n_z = 1 - \ddot{z} / g \tag{17}$$

Combining Eq. (17) with the definition of glideslope angle γ, the time derivative of the glideslope angle can be expressed in terms of the normal load factor n_z , which is:

$$\dot{\gamma}=\frac{\dot{V}\dot{z}-Vg(1-n_z)}{V^2\cos\gamma} \tag{18}$$

According to the aerodynamic performance of coaxial rigid rotor, its maximum normal load factor could achieve 2.2 [10, 25]. Thus, the load factor distribution that relates to the level 1 standard set is shown in Fig. 12.

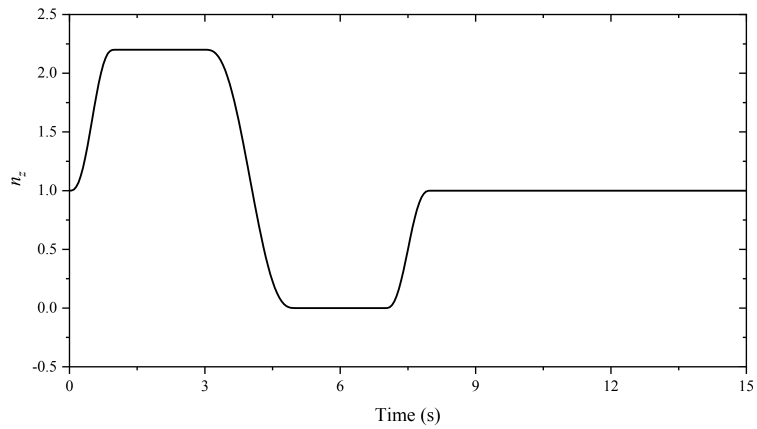


Fig. 12 Level 1 Load Factor throughout the Pull-up & Push-over MTE

Therefore, the manoeuvre is split into six sections and fifth-order polynomials are formed to describe the load factor distribution at each section. The polynomial should guarantee the value of the load factor at every time points satisfy the requirement and the transition between each of the sections will be smooth.

Except for the boundary condition of Eq. (18), there are three additional boundary conditions required for the helicopter. Firstly, the velocity in this MTE is assumed to be the function of the glideslope angle, which is based on the balance of energy:

$$\dot{V}=-g\sin\gamma \tag{19}$$

Also, the boundary condition of track angle and heading angle should be fixed at zero across the manoeuvre:

$$\chi=0 \tag{20}$$

$$\dot{\Psi}=0 \tag{21}$$

Eqns. (18~21) compose the boundary condition of this MTE. However, the coaxial compound helicopter has its redundant control input that should be mentioned. The LOS control strategy is the research objective in this article and is discussed below. The propeller control in this manoeuvre is set to keep its collective at the trim value. The heading control strategy follows the strategy of Eqns. (6~7). The variable rotor speed is defined by Eq. (5). Fig. 13 is the calculation results of this MTE using the inverse simulation method. In this calculation process, the LOS control strategy (differential lateral cyclic) is fixed at the trim value.

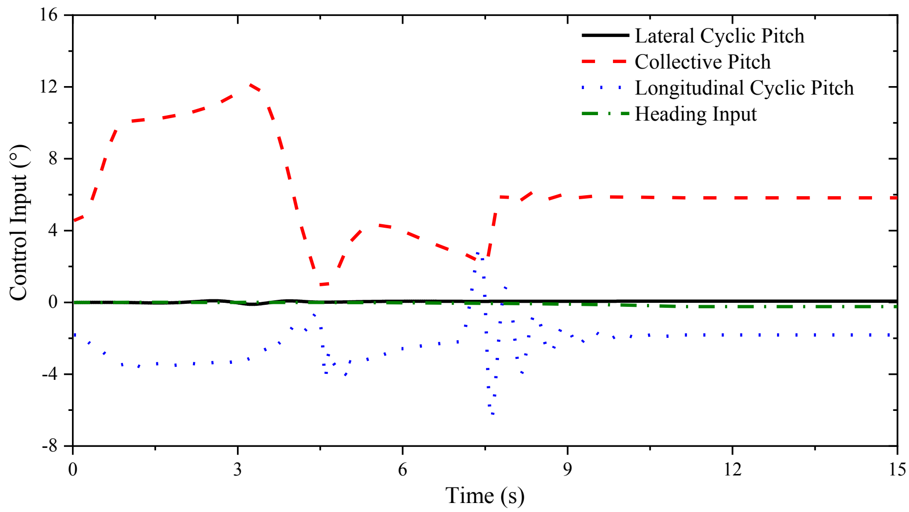


Fig. 13 The Pull-up & Push-over MTE Control Input (θ_{dc} Fixed)

Based on the results, this coaxial compound helicopter could achieve this MTE with differential lateral cyclic pitch fixed. However, there is a significant oscillation of the control input in longitudinal and collective pitch. Compared with the results of other types of helicopter [44], the oscillation is more severe, and it is due to the LOS characteristics. When the differential lateral cyclic pitch is fixed, LOS alters significantly during the manoeuvre due to the change of the collective pitch. The above analysis have already shown that LOS induces extra coupling in longitudinal and normal axis of the helicopter.

On the other hand, the LOS value influences the aerodynamic performance of the coaxial rigid rotor and further changes the power required. Thus, the LOS control strategy for the Pull-up & Push-over MTE can be set to consider the performance analysis above, rather than simply fixing it at the trim value. The proposed LOS control strategy with time is shown in Fig.14. This strategy is scheduled to follow the normal loading changes for the coaxial compound helicopter to have good performance.

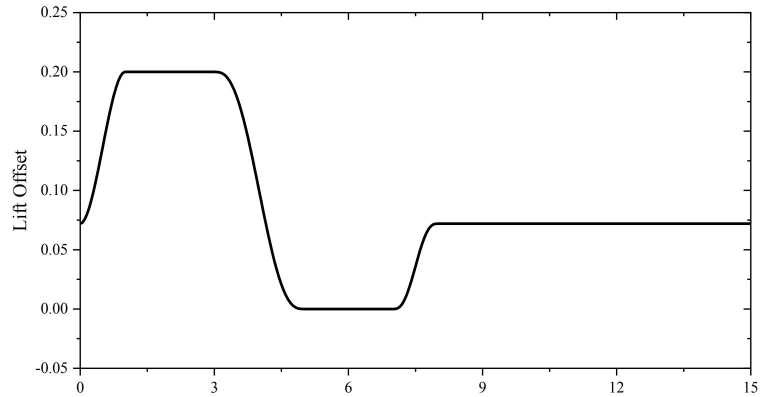


Fig. 14 LOS Control Strategy of Pull-up & Push-over MTE (Correspond to Load Factor)

Based on the LOS control strategy in Fig. 14, the new control inputs of the Pull-up & Push-over MTE are given in Fig. 15. The differential lateral cyclic pitch results are also included in this figure.

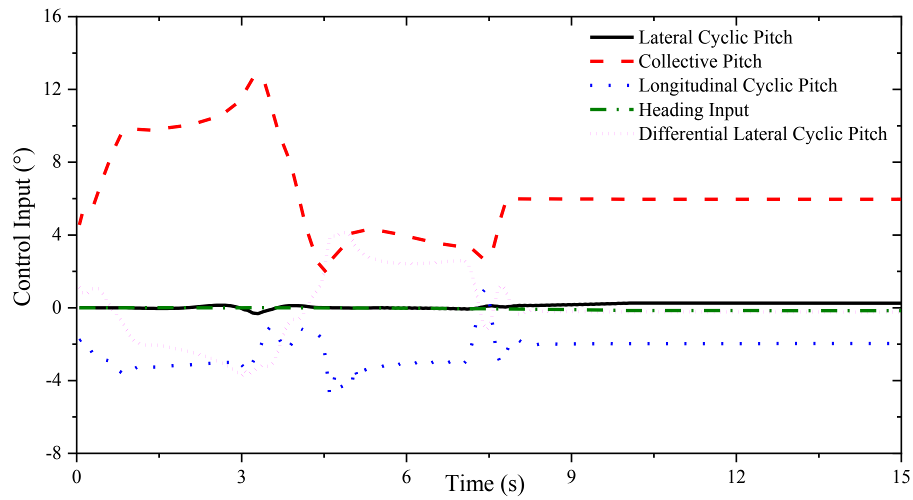


Fig. 15 The Pull-up & Push-over MTE Control Input (LOS Setting Based on Fig. 12)

Utilizing the LOS control strategy in Fig.14, the helicopter achieves this MTE without too much oscillation compared with the original results. The collective pitch and the longitudinal cyclic pitch are no longer oscillate around 6s to 10s time period. It demonstrates that LOS is one of the reasons that induce the severe oscillation in this MTE. Also, the use of the differential lateral cyclic pitch would reduce the collective pitch inputs due to the coupling between the lift and the differential lateral cyclic. Fig.16 shows the power consumption comparison between the original LOS strategy (θ_{dc} fixed) and the proposed LOS control strategy.

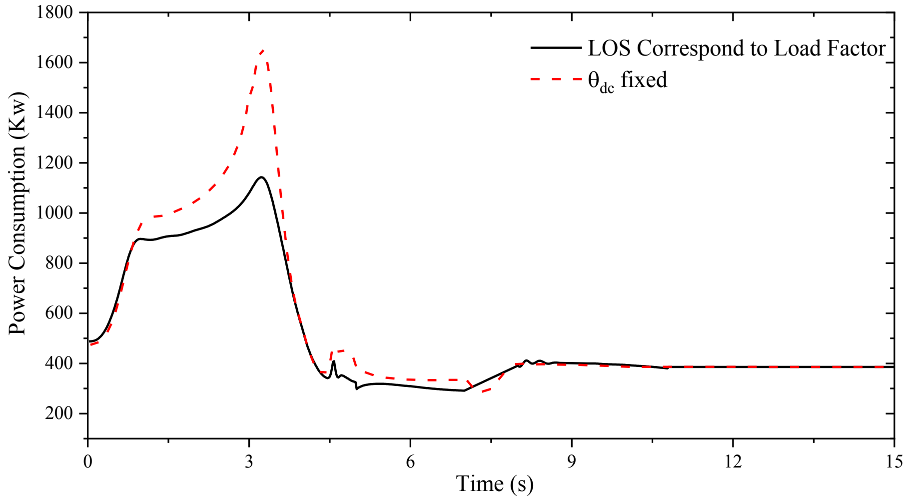


Fig. 16 Power Consumption Comparison of Pull-up & Push-over MTE

It shows clearly that the proposed LOS control strategy could reduce the power consumption, especially when the load factor is relatively high (from 1s to 3s). The re-scheduled LOS value would further improve the rotor efficiency, which in turn reduce the power consumption.

B. Transient Turn MTE

The transient turn MTE is designed to assess the handling qualities related to heading and lateral axis in high speed flight. The mathematical description of the Transient Turn MTE [42-43] can be defined by Eqns. (22-25) according to the requirement on ADS-33E-PRF.

$$\dot{V} = 0 \tag{22}$$

$$\dot{\gamma} = 0 \tag{23}$$

$$\beta_{sa} = 0 \tag{24}$$

$$\dot{\chi} = f(t) \tag{25}$$

where β_{sa} is the sideslip angle. The next step is to define the track angle distribution $\dot{\chi} = f(t)$ throughout the manoeuvre by the related boundary conditions. Fig.17 is the track angle derivative $\dot{\chi}$ distribution which relates to the Level 1 requirement set in the specification.

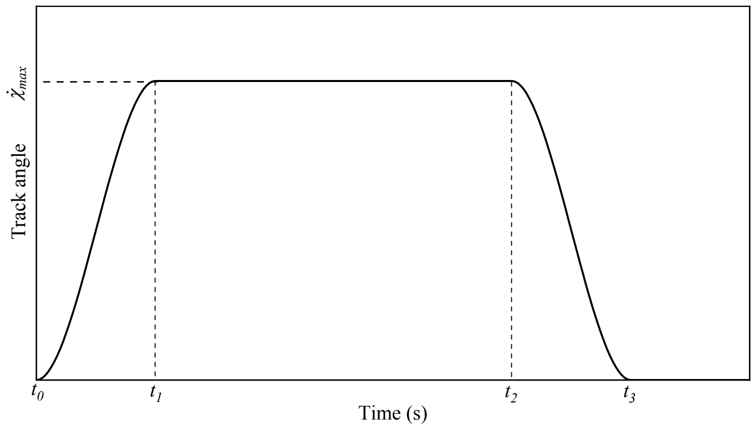


Fig. 17 Level 1 track angle derivative throughout the Transient Turn MTE

Therefore, the track angle derivative in each section of this MTE can be described using the fifth-order polynomials, similar to the method used in the Pull-up & Push-over MTE. Fig.18 shows the control inputs for the Transient Turn MTE using inverse simulation method. In this calculation, the redundant control setting is same as that of the Pull-up & Push-over MTE, and LOS control strategy (differential lateral cyclic) is firstly fixed at the trim value.

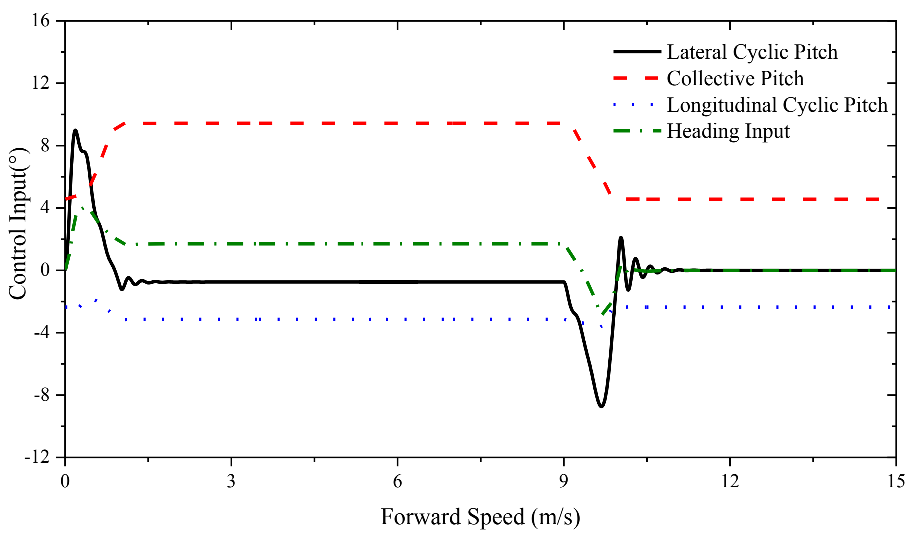


Fig. 18 The Transient Turn MTE Control Input (θ_{dc} fixed)

The results show that the coaxial compound helicopter could achieve the Transient Turn MTE with differential lateral cyclic pitch fixed, and almost all control input is smooth. Only the lateral cyclic pitch and the rudder deflection have a little oscillation at the end of this manoeuvre. As in the previous analysis, LOS has very little influence on the lateral channel since the coupling rolling moments are self-balanced. On the other hand, the collective pitch input is relatively high during the Transient Turn MTE. The re-design of the LOS control strategy may be a useful tool to optimize the power consumption during this MTE. Therefore, considering the Pull-up & Push-over MTE investigation, the LOS control strategy is set based on the track angle velocity, as shown in Fig. 19.

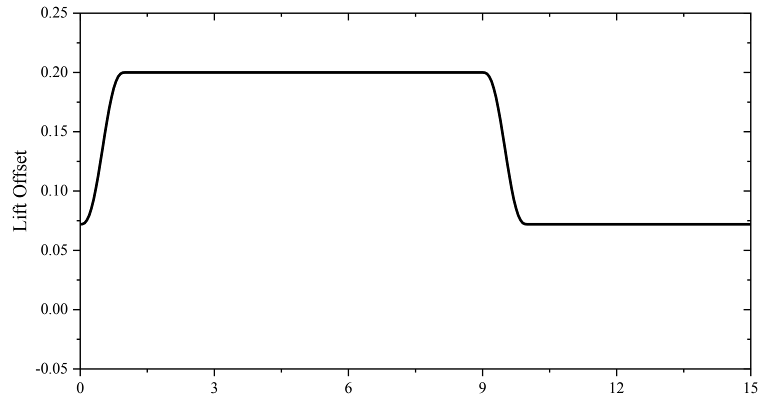


Fig. 19 LOS Control Strategy of Transient Turn MTE (Correspond to Track Angle Velocity)

According to this LOS control strategy, the control input results of the Transient Turn MTE are shown in Fig.20, which includes the differential lateral cyclic pitch.

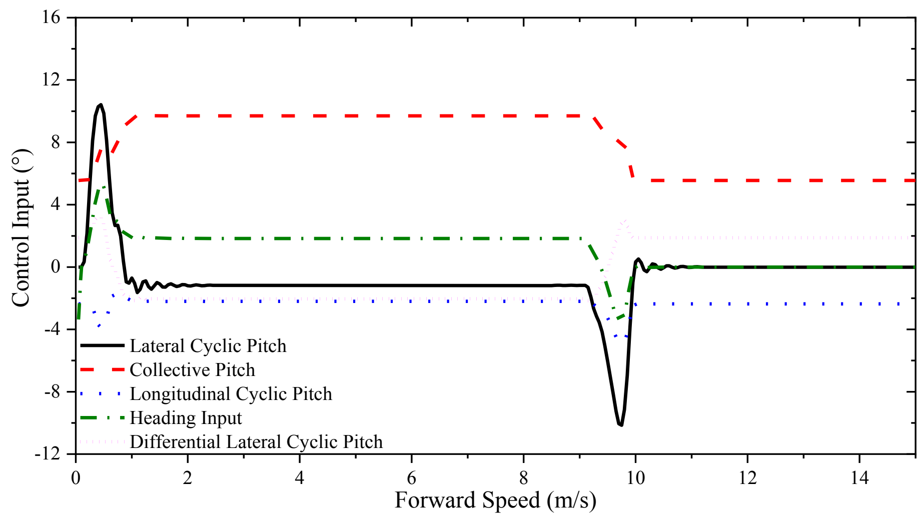


Fig. 20. The Transient Turn MTE Control Input (LOS Correspond to Track Angle Velocity)

Comparing Fig.20 and Fig.18, there are two differences in control input results. Firstly, the amplitude of the lateral cyclic pitch is slightly more when the LOS strategy is adopted. According to Eq. (8), the LOS value links to the rolling hub moments provided by the rotor. In other words, it would limit the capability of the rotor in providing the rolling moment when LOS has to maintain a given value, which would increase the lateral cyclic pitch displacements. In addition, the oscillation of collective pitch and the lateral cyclic pitch overall is smoother compared with Fig. 18. It demonstrates that an appropriate LOS control strategy has the ability to improve the stability in manoeuvreing flight. Fig. 21 shows the power consumption comparison results, in which the power required is significantly reduced when LOS is re-scheduled as it could guarantee LOS in the range where the rotor has a higher performance.

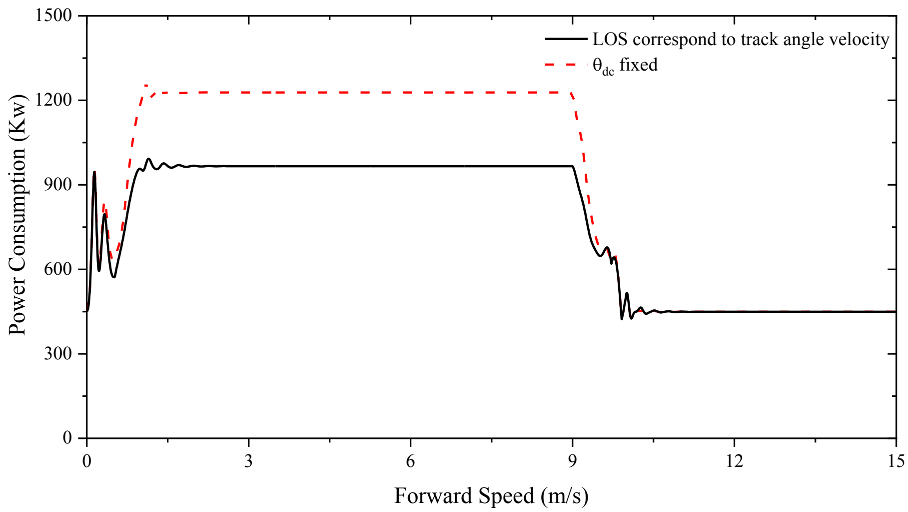


Fig. 21 Power Consumption Comparison of Transient Turn MTE

As illustrated above, LOS value not only influences the power required in manoeuvring flight, but also it changes the control inputs during the manoeuvre due to the coupling effect of LOS. Therefore, the LOS optimization should be an effective method to improve the flight dynamics characteristics of the coaxial compound helicopter during the manoeuvre.

VIII.Conclusion

This article utilized a flight dynamics model to investigate the influence of Lift Offset (LOS) on the flight dynamics characteristics of the coaxial compound helicopters. This includes the trim features, power consumption, stability, controllability, and manoeuvre characteristics. The results allow the following conclusions to be drawn:

1) LOS influences the trim characteristics of the coaxial compound helicopter. Increasing LOS would reduce the trim collective pitch and longitudinal cyclic pitch because of the improvement of rotor efficiency and the effect of the flapping motion. In hover and low speed forward flight, a relatively high LOS adds the profile drag of the coaxial rotor and consequently results in the increment in trim propeller collective.

2) LOS changes the power consumption across the speed range and can be a potential way to improve the maximum speed of the helicopter. The LOS value should increase with forward speed to maintain higher performance.

However, the LOS control strategy is not only related to the forward speed, but also affected by the other factors, such as the gross weight.

3) The velocity, incidence, and dihedral effect stability derivatives are influenced by LOS. The velocity and incidence stabilities are improved by reasonable LOS strategy. However, the dihedral effect stability decreases with LOS as it reduces the incidence at the 0-degrees and 180-degrees azimuth angle on the rotor disc. On the other hand, LOS have rarely influence on the heading stability derivatives.
4) The influence of LOS on the on-axis derivatives is mainly due to its effect on the rotor aerodynamic performance. Severe coupling between the rolling moment and the differential collective inputs is observed. In addition, the control of LOS would induce additional rotor lift and pitching moment to coaxial compound helicopters.
5) Using inverse simulation, the Pull-up & Push-over MTE and Transient Turn MTEs were analyzed to investigate the influence of LOS on manoeuvre characteristics. LOS affects the control input oscillation during the manoeuvre and an appropriate LOS control strategy can be used to prevent it. LOS control strategy can also reduce the power consumption by improving the rotor efficiency during the manoeuvre.

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